Technical Comments

Comments on "Performance Characteristics of a Magnetic Annular Arc"

Raffaele Giovanelli*
Politecnico di Milano, Milano, Italy

In its February 1966 issue, the AIAA Journal published an article by Patrick and Schneiderman[1] in which is explained a simplified theory about the relationship existing between the V-B characteristics of the MAARC arc and the nature of several propellants, various mass flows, and input power. The assumption made and the conclusions outlined are in accordance with the first analysis, performed by Alfvén[2] and discussed by Petschek and others,[2] about some Fahleson[3] experimental results on a "homopolar" device. In the development of the theoretical explanation of the existence of a critical speed of ions in the ionization region, no differentiation was made between the current density j, which is used in the energy equation:

$$n(\epsilon_{d_1i} + C_pT + mU^2/2) = P_{\text{in}} = jE \tag{1}$$

and the current density used in the momentum equation:

$$jB = \dot{n}(mU) \tag{2}$$

where $\dot{n}$ is the incoming number density flow rate (per unit area); ϵ_{d_1i} is the energy per incoming particle due to dissociation and ionization; T is the temperature at the end of ionization region; U is the velocity of the exiting ions; j is the current density; and E is the local electric field.

If δz is the depth of the ionization region where the incoming room-temperature gas starts carrying current (see Fig. 1), and if $\delta\dot{n}$ is the number of particles dissociated and ionized per unit cross-sectional area and per unit time, $\delta z \cdot jE$ will be the electrical input power to the gas volume: $1 \cdot 1 \cdot \delta z$. So the Eq. (1), with the assumption $\epsilon_{d_1i} \gg C_pT$ can be rewritten in the form:

$$\delta\dot{n}[\epsilon_{d_1i} + (mU^2/2)] = \delta z \cdot j_rE_r \tag{3}$$

where the radial current density j_r is given by $j_r = (j_{er} + j_{ir})$: the sum of electron current† and the ion current in the radial direction.

In the ionization depth, less than the distance an ion must diffuse to transfer its momentum to neutral background gas, if is correct that the ions will drift with nearly the local E/B velocity, this is also true for electron drift. So, for the null contribution of the electron gas to the azimuthal component in the momentum equation (low pressure field), it is right to introduce, in Eq. (2), the current density due only to ions, rather than the total current j. In the energy equation (1) however, it is correct to introduce the total current density to obtain the input power per unit volume. A strong velocity difference, due to the external applied voltage of the MAARC electrodes, exists between electrons and ions in the radial motion. So, by the energy exchange (see Fig. 1) between electrons, ions and neutral gas in the radial motion, is justified the total current j_r in Eqs. (1) and (3). Equation (2), with the new assumption, must be written in the form:

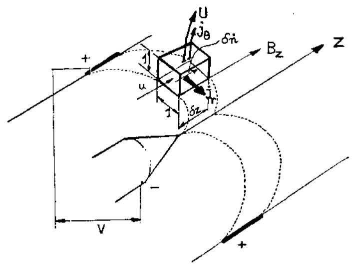

Fig. 1 Notations for energy and momentum equations of ionization region of MAARC arc. Local orthogonal coordinates are used (instead of cylindrical ones) to represent the volume element $1 \cdot 1 \cdot \delta z$ of ionization region.

$$\delta z \cdot j_{i,r}B_z = \delta\dot{n} \cdot (m_iU) \tag{4}$$

Following the Cowling[4] procedure for the partially ionized gases from a solution presented by Brunner,[5] we obtain for the total current density in a partially ionized gas the following expression:

$$\mathbf{j} = \sigma_0 \frac{1 + b}{(1 + b)^2 + (\omega_e\bar{\tau}_e)^2} \times \mathbf{E} - \sigma_0 \frac{w_e\bar{\tau}_e}{(1 + b)^2 + (\omega_e\bar{\tau}_e)^2} \mathbf{E} \times \frac{\mathbf{B}}{B} \tag{5}$$

where

$$b = 2f^2\omega_i\tau_{in}w_e\bar{\tau}_e$$

where the term $[b/(1 + 2b)\sigma_0(\mathbf{E} \cdot \mathbf{B}/B^2)]\mathbf{B}$ is omitted because the scalar product $\mathbf{E} \cdot \mathbf{B}$ can be taken zero. From Eqs. (6–21),[4] neglecting the term containing the electron pressure p_e, we can obtain an expression for $\mathbf{j}_i$, the current density due to the ions motion only:

$$\mathbf{j}_i = \frac{f(\mathbf{j} \times \mathbf{H}) + k_eH\mathbf{j}}{H(k_e + k_i)} f \qquad \text{where:} \quad \begin{aligned} k_e &= 1/(\omega_e\tau_{en}) \\ k_i &= 1/(2\omega_i\tau_{\text{in}}) \end{aligned} \tag{6}$$

$$\sigma_0 = n_ee^2\tau_e/m_e \qquad \omega_e = eB/m_e$$

$$f = n_a/(n_a + n) \qquad \text{fraction of atoms not ionized}$$

$$\bar{\tau}_e = [(1/\tau_{ei}) + (1/\tau_{en})]^{-1}$$

For the r component we have:

$$j_{ir} = \frac{f^2j_\theta + fk_ej_r}{k_e + k_i} \tag{7}$$

The electron radial current can be obtained subtracting j_{ir} of (7) from j_r that is derived from Eq. (5):

$$j_r = E_r\sigma_0 \frac{(1 + b)}{(1 + b)^2 + (\omega_e\bar{\tau}_e)^2} \tag{8}$$

$$j_{er} = j_r - j_{ir} = \frac{1 + b}{(1 + b)^2 + (\omega_e\bar{\tau}_e)^2}\sigma_0E_r - \frac{f^2j_\theta + fk_ej_r}{k_e + k_i} = \sigma_0E_r \frac{1 + b}{(1 + b)^2 + (\omega_e\bar{\tau}_e)^2} \times \left[1 - \frac{fk_e}{(k_e + k_i)}\right] - \sigma_0E_r \frac{\omega_e\bar{\tau}_e}{(1 + b)^2 + (\omega_e\bar{\tau}_e)^2} \frac{f}{(k_e + k_i)} \tag{9}$$

From Eq. (4) the current j_{ir} is: $j_{ir} = \dfrac{\delta\dot{n}}{\delta z} \cdot \dfrac{m_iU}{B_z}$, substitution for j_{ir} into energy equation (3) gives:

$$\left[\frac{m_iU}{B_z} \frac{\delta\dot{n}}{\delta z} + j_{er}\right] E_r = \frac{\delta\dot{n}}{\delta z} [\epsilon_{d_1i} + m_iU^2/2] \tag{10}$$

Received November 29, 1966.

* Engineer, past Assistant, Instituto di Fisica Tecnica e Macchine,

† The radial component of the electron current density j_{er} is also due to the ionization collisions in the ionization region. Without a sufficiently high frequency of the ionization collision, no stationary plasma can exist in a moving gas; of course the collisions are responsible for all of the diffusion process across the magnetic field.

But for low pressure field was assumed: $U \cong E_r/B_z$; so Eq. (10) becomes:

$$U/2 + \frac{j_{er}E_r}{m_iU(\delta\dot{n}/\delta z)} = \epsilon_{d_1i}/(m_iU)$$
$$U^2 = 2\left[\epsilon_{d_1i}/m_i - \frac{j_{er}E_r}{m_i(\delta\dot{n}/\delta z)}\right] \tag{11}$$

The expression (11) for ion velocity U differs from the corresponding expression obtained by Patrick and Schneiderman[1] for the term $-j_{er}E_r/[m_i(\delta\dot{n}/\delta z)]$. The radial component of the current density j_{er} due to electron motion [Eq. (9)] is strongly affected by the orthogonal magnetic field and its contribution to the total current j_r can be less than the ion radial current j_{ir}. From the data reported by Fahleson[3] on the existence of a critical velocity v_c of ions, and following the calculations performed by Alfvén,[2] a difference of some electron volts is found between the measured and the calculated ion energy. This difference shows that the measured value of v_c is less than the calculated value, taking into account the total current density in both the momentum and the energy equations. So could be justified the existence of the term $-j_{er}\ E_r/[m_i(\delta\dot{n}/\delta z)]$ in Eq. (11) obtained with ion current in the momentum equation (4).

References

[1] Patrick, R. M. and Schneiderman, A. M., "Performance characteristics of a magnetic annular arc," AIAA J. 4, 283–290 (1966).

[2] Alfvén, H., "Collision between a nonionized gas and a magnetized plasma," Rev. Mod. Phys. 32, 710–713 (1960).

[3] Fahleson, U. V., "Experiments with plasma moving through neutral gas," Phys. Fluids 4, 123–127 (1951).

[4] Cowling, T. G., *Magnetohydrodynamics* (Interscience Publishers Inc., New York, 1957), Chap. VI, pp. 105–109.

[5] Brunner, M. J., "The effects of hall and ion slip on the electrical conductivity for magnetohydrodynamic re-entry vehicle application," J. Heat Trans., 177–184 (1962).

Reply by Authors to R. Giovanelli

R. Patrick* and A. Schneiderman†
Avco Everett Research Laboratory, Everett, Mass.

The argument presented by the authors was meant only to illustrate the kinds of arguments that are used in discussing the critical velocity, and was intended only to make plausible the appearance of a critical velocity and indicate the relevance of the original work on this subject by Alfvén and others. The subject of the critical veloicity is by no means closed, and a large literature[1–22] has evolved, based upon either the macroscopic balances of the kind used by Petschek,[1] or more microscopic effects involving the electron energy balance as used by Drobyshevsky.[2] The approach presented by the authors has been further amplified by Lin.[3] However, the basic assumption of this approach is that the coupling to the neutrals is small and neutral effects do not play an important role in the ionized components, energy, and momentum balance. This assumption is not agreed to by others, including Lehnert[4] and Block.[5] Lin shows that acceptance of this assumption implies that the ions, because of their larger mass, must take up the momentum caused by the $j \times B$ forces, although the electrons can absorb some of the energy. This leads to the use of the total current in the ion momentum equation but with an electron energy term included in the ion energy equation.

It is not clear how this work relates to the effect pointed out in the aforementioned technical comment. However, this approach in general is valid only for weak coupling between the ions and the neutrals $\omega_i\tau_{in} \gg 1$, in which case the electron current is negligible [see previously mentioned Eq.(9)], and the change in the critical velocity pointed out in the comment goes to zero.

Because the basic assumptions on which the whole approach is based are subject to question and because granted that the assumptions are correct, the change in critical velocity vanishes, the authors believe that the work in this note is not significant to the problem of the critical velocity.

Received January 23, 1967.
* Principal Research Scientist.
† Research Associate. Associate Member AIAA.

References

[1] Alfvén, H., "Collision between a non-ionized gas and a magnetized plasma," Rev. Modern Phys. 32, 710 (1960).

[2] Drobyshevsky, E. M., "The volt-ampere characteristics of a homopolar cell," Soviet Physics-Technical Physics 8, 903 (1964), translated from Zhurnal Tekhnicheskoi Fiziki 33, 1210 (1963).

[3] Lin, S. C., "On the limiting velocity for a rotating plasma," Phys. Fluids 4, 1277 (1961).

[4] Lehnert, B., "Ionization process of a plasma," Phys. Fluid 9, 774 (1966).

[5] Block, L., "Critical velocity of partly ionized gases," presented to American Physical Society, San Diego, Calif. (November 1963).

[6] Powers, W. E. and Patrick, R. M., "Magnetic annular arc," Phys. Fluids 5, 1196 (1962).

[7] Chubb, D. L. and Seikel, G. R., "Basic studies of a low density Hall current ion accelerator," NASA TN D-3250.

[8] Bowditch, D. N., "Investigation of the discharge and exhaust beam of a small arc plasma thrustor," NASA TM X-52175.

[9] Burlock, J., Brockman, P., Hess, R. V., and Brooks, D. R., "Measurement of velocities and acceleration mechanism for coaxial Hall accelerators," AIAA Paper 66-196 (1966); also AIAA J. 5, 558–561 (1967).

[10] Drobyshevsky, E. M. and Chekmarev, I. B., "Positive column in a rotating homopolar," Soviet Phys. Tech. Phys. (USA) 10, 1206–1212 (1966).

[11] Adams, V. W., "The influence of gas streams of magnetic fields on electric discharges," Ministry of Aviation, England, CP 743 (1964).

[12] Workman, J. B., "Insulator ablation in magnetic piston shock tubes," Phys. Fluid 12, 2162 (1965).

[13] Wilcox, J. M., Pugh, E. R., Dattner, A., and Eninger, J., "Experimental study of the propagation of an ionizing wave in a coaxial plasma gun," Phys. Fluids 7, S51 (1964).

[14] Patrick, R. M. and Pugh, E. R., "An experimental and theoretical study of MHD ionizing fronts," Avco Everett Research Lab. Rept. 188 (1964).

[15] Wilcox, J. M., Baker, W. R., Boley, F. I., Cooper, III, W. S., DeSilva, A. W., and Spillman, G. R., "Devices for generating highly-ionized hydrogen plasma," J. Nuclear Energy, Part C, 4, 337 (1962).

[16] Brennan, M. H., Brown, I. G., Millar, D. D., and Watson-Munro, C. N., "Some experimental observations of the characteristics of hydromagnetic ionizing fronts," J. Nuclear Energy Part C, 5, 229 (1963).

[17] Fahleson, U. V., "Experiments with plasma moving through neutral gas," Phys. Fluids 4, 123 (1961).

[18] Angerth, B., Block, L., Fahleson, U., Soop, K., "Experiments with partly ionized rotating plasmas," Nuclear Fusion, Supplement, Part I, 39 (1963).

[19] Alfvén, H., *On the Origin of the Solar System* (Oxford University Press, London, 1954).

[20] Alfvén, H. and Wilcox, J. M., "On the origin of the satellites and the planets," Astrophys. J. 136, 1016 (1962).

[21] Chodura, R., "Zum Zündmechanismus einer Theta-Pinch-Entladung," Z. Naturforschg. 19a, 679 (1964).

[22] Lehnert, B., Phys. Fluids 9, 774–779 (1966).